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Non-thermal production of dangerous relics in the early universe

Gian Francesco Giudice, Antonio Riotto and Igor Tkachev

CERN Theory Division, CH-1211 Geneva 23, Switzerland.
E-mail: riotto@nxth04.cern.ch, Gian.Giudice@cern.ch, Igor.Tkachev@cern.ch

Abstract: Many models of supersymmetry breaking, in the context of either supergravity or superstring theories, predict the presence of particles with weak scale masses and Planck-suppressed couplings. Typical examples are the scalar moduli and the gravitino. Excessive production of such particles in the early Universe destroys the successful predictions of nucleosynthesis. In particular, the thermal production of these relics after inflation leads to a bound on the reheating temperature, $T_{RH} \lesssim 10^9$ GeV. In this paper we show that the non-thermal generation of these dangerous relics may be much more efficient than the thermal production after inflation. Scalar moduli fields may be copiously created by the classical gravitational effects on the vacuum state. Consequently, the new upper bound on the reheating temperature is shown to be, in some cases, as low as 100 GeV. We also study the non-thermal production of gravitinos in the early Universe, which can be extremely efficient and overcome the thermal production by several orders of magnitude, in realistic supersymmetric inflationary models.

Keywords: Cosmological Phase Transitions, Neutrino and Gamma Astronomy.

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Contents

1. Introduction

One of the problems facing Planck-scale physics is the lack of predictivity for low-energy phenomena. Nonetheless, some information on high-energy physics may be inferred indirectly through its effects on the cosmological evolution of the early Universe. If we require that the successful predictions of big-bang nucleosynthesis are not significantly modified and that the energy density of stable particles does not overclose the Universe, severe constraints are imposed on the properties of a large class of supersymmetric theories.

In $N = 1$ supergravity models [1], supersymmetry is broken in some hidden sector and gravitational-strength interactions communicate the breaking down to the visible sector. In these models there often exist scalar and fermionic fields with masses of the order of the weak scale and gravitational-strength couplings to ordinary matter. In the following, we will generically refer to them as *gravitational relics* X. If produced in the early Universe, such quanta will behave like nonrelativistic matter and decay at very late times, eventually dominating the energy of the Universe until it is too late for nucleosynthesis to occur (in the case of scalar fields, long-lived coherent oscillations with large amplitudes of the zero mode can pose even a more serious problem [2]). Typical examples of gravitational relics are the spin-3/2 gravitino – the supersymmetric partner of the graviton – and the moduli fields – the quanta of the scalar fields which parametrize supersymmetric flat directions in moduli space and seem almost ubiquitous in string theory.

In string models massless fields exist in all known ground states and parametrize the continuous vacuum degeneracies characteristic of supersymmetric theories. Possible examples of gravitational relics in string theory are the dilaton whose vacuum expectation value parametrizes the strength of the gauge forces, and the massless

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scalar (and fermionic superpartner) gauge singlets parametrizing the size of the compactified dimensions. In general, these fields are massless to all orders in perturbation theory in the exact supersymmetric limit and become massive either because of non-perturbative effects or because of supersymmetry-breaking contributions. In the usual scenarios in which supersymmetry breaking occurs at an intermediate scale $M_S = \mathcal{O}(\sqrt{M_{\rm P} M_W})$, moduli acquire masses of the order of the weak scale and, because of their long lifetime, pose a serious cosmological problem [3]. The problem becomes even more severe in models in which supersymmetry is broken at lower scales, since the moduli are lighter than M_W and their lifetime becomes exceedingly long. On the other hand, if supersymmetry breaking occurs at a scale larger than M_S and its effect on the observable sector is screened, the cosmological problem of gravitational relics is relaxed. In particular, this is the case of the recently-proposed models with anomaly-mediated supersymmetry-breaking [4], in which moduli and gravitinos can avoid cosmological difficulties [5].

Gravitational relics X can be produced in the early Universe because of thermal scatterings in the plasma. The slow decay rate of the X-particles is the essential source of cosmological problems because the decay products of these relics will destroy the ^{4}He and D nuclei by photodissociation, and thus successful nucleosynthesis predictions [6, 7]. The most stringent bound comes from the resulting overproduction of D + ^{3}He; this requires that the X-abundance relative to the entropy density at the time of reheating after inflation should satisfy [8]

$$\frac{n_X}{s} \lesssim 10^{-12}. \tag{1.1}$$

(The exact bound depends upon the X mass, and here we have assumed $M_X \sim$ TeV.) Neglecting any initial number density, the Boltzmann equation for the number density of gravitational relics during the thermalization stage after inflation is

$$\frac{dn_X}{dt} + 3Hn_X \simeq \langle \sigma_{\rm tot} |v| \rangle n^2_{\rm light}, \tag{1.2}$$

where $\sigma_{\rm tot} \propto 1/M_{\rm P}^2$ is the total production cross section and $n_{\rm light} \sim T^3$ represents the number density of light particles in the thermal bath. The number density n_X at thermalization is obtained by solving eq. (1.2),

$$\frac{n_X}{s} \simeq 10^{-2}\,\frac{T_{RH}}{M_{\rm P}}. \tag{1.3}$$

Comparing eqs. (1.1) and (1.3), one obtains an upper bound on the reheating temperature after inflation [7]

$$T_{RH} \lesssim (10^8 - 10^9)\ {\rm GeV}. \tag{1.4}$$

If $T_{RH} \sim M_{\rm GUT} \sim 10^{16}$ GeV, gravitational relics are inconsistent with nucleosynthesis. Moreover, if the initial state after inflation is free from gravitational relics, the

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reheating temperature in eq. (1.4) is too low to allow for the creation of superheavy GUT bosons that can eventually produce the baryon asymmetry [9].

The goal of this paper is to point out that gravitational relics can be created with dangerously large abundances by *non-thermal* effects.

One possibility is represented by the non-thermal effect of the classical gravitational background on the vacuum state during the evolution of the Universe. The particle creation mechanism is similar to the inflationary generation of gravitational perturbations that seed the formation of large scale structures. However, the quantum generation of energy density fluctuations from inflation is associated with the inflaton field which dominated the mass density of the Universe, and not a generic, sub-dominant scalar or fermionic field. Gravitational particle creation has recently been employed to generate non-thermal populations of very massive particles [10, 11]. In particular, the desired abundance of superheavy dark matter particles may be generated during the transition from the inflationary phase to a matter/radiation dominated phase as the result of the expansion of the background spacetime acting on vacuum quantum fluctuations of the dark matter field. Contrary to the generation of density perturbations, this mechanism contributes to the homogeneous background energy density that drives the cosmic expansion, and is essentially the familiar "particle production" effect of relativistic field theory in external fields.

Another possibility is represented by the non-thermal effects occuring right after inflation because of the rapid oscillations of the inflaton field(s). Gravitino production is an interesting example of this kind.

The paper is organized as follows. In sect. 2 we show that the gravitational production of scalar moduli which are minimally coupled to gravity may be much more efficient than the one activated by thermal scatterings during reheating. Our findings lead to extremely stringent constraints on the reheating temperature after inflation. In sect. 3 we study the non-thermal production of gravitinos in the early Universe, showing that the helicity-1/2 part of the gravitino can be efficiently excited during the evolution of the Universe. This leads to a copious and dangerous generation of gravitinos in realistic supersymmetric models of inflation. Previous studies [12] of non-thermal production of gravitinos have considered only the subleading effects of the helicity-3/2 components. However, a thorough study of the complete gravitino equations has very recently appeared [13]. Although the derivation of the gravitino equations presented in our paper is different from that of ref. [13], our final results agree fully with the analysis of ref. [13]. Finally, sect. 4 contains our conclusions.

2. Gravitational production of scalar moduli

Let us describe here the basic physics underlying the mechanism of gravitational scalar particle production.

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We start by canonically quantizing the action of a generic scalar massive field X which, at the end, will be identified with a modulus field. In the system of coordinates where the line element is given by $ds^2 = dt^2 - a^2(t)d\vec{x}^2$, the action is

$$S = \int dt \int d^3x \, \frac{a^3}{2} \left[\left(\frac{dX}{dt} \right)^2 - \frac{(\vec{\nabla} X)^2}{a^2} - M_X^2 X^2 - \xi R X^2 \right], \tag{2.1}$$

where R is the Ricci scalar. After transforming to conformal time coordinate, where the line element is $ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2)$, we use the field mode expansion

$$\begin{aligned} X = \int \frac{d^3k}{(2\pi)^{3/2} a(\eta)} \Big[& a_k h_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \\ & + a_k^\dagger h_k^*(\eta) e^{-i\vec{k}\cdot\vec{x}} \Big]. \end{aligned} \tag{2.2}$$

Since the creation and annihilation operators obey the commutator relations

$$\left[a_{k_1}, a_{k_2}^\dagger \right] = \delta^{(3)}(\vec{k}_1 - \vec{k}_2)$$

, we obtain the normalization condition $h_k \dot{h}_k^* - \dot{h}_k h_k^* = i$ (henceforth, the dots over functions denote derivatives with respect to η). The resulting mode equation is

$$\ddot{h}_k(\eta) + \omega_k^2(\eta) h_k(\eta) = 0, \tag{2.3}$$

where

$$\omega_k^2 = k^2 + M_X^2 a^2 + (6\xi - 1)\frac{\ddot{a}}{a} \, . \tag{2.4}$$

The parameter ξ is $1/6$ for conformal coupling and 0 for minimal coupling. The equation of massless conformally coupled quanta reduces to the equation in flat space-time and, in this case, there is no particle production. When $M_X \neq 0$ or $\xi \neq 1/6$, conformal invariance is broken and particles are created. In the following we will be especially interested in the case of scalar particles with non-conformal coupling. In particular, the case of minimal coupling ($\xi = 0$) is certainly of physical interest. Just to give one example, in the toroidal compactification of the type IIB string theory, the modulus field describing the volume of the internal compactified dimensions is minimally coupled to gravity in the four dimensional effective action.

The differential equation (2.3) can be solved once the boundary conditions are supplied. Since the annihilation operator is just a coefficient of an expansion in a particular basis, fixing the boundary conditions is equivalent to fixing the vacuum. We choose the initial conditions as

$$h_k(\eta_0) = \omega_k^{-1/2}, \quad \dot{h}_k(\eta_0) = -i\omega h_k(\eta_0), \tag{2.5}$$

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corresponding to a vanishing particle density at $\eta = \eta_0$. To obtain the number density of the produced particles, we perform a Bogolyubov transformation from the vacuum mode solution with the boundary condition at $\eta = \eta_0$ into the one with the boundary condition at $\eta = \eta_1$ (any later time at which the particles are no longer being created). Defining the Bogolyubov transformation as $h_k^{\eta_1}(\eta) = \alpha_k h_k^{\eta_0}(\eta) + \beta_k h_k^{*\eta_0}(\eta)$ (the superscripts denote where the boundary condition is set), we obtain the following number density of produced particles:

$$n_X(\eta_1) = \frac{1}{2\pi^2 a^3(\eta_1)} \int_0^\infty dk\, k^2 |\beta_k|^2. \tag{2.6}$$

One should note that the number operator is defined at η_1 while the quantum state (approximated to be the vacuum state) defined at η_0 does not change in time in the Heisenberg representation.

Let us now discuss the structure of the mass term of the X scalar particle. The flat directions in the supersymmetric potential, corresponding to massless X particles, are lifted by supersymmetry breaking and nonrenormalizable effects in the superpotential. An important observation for us is that in the early Universe global supersymmetry is broken (for instance because of the false vacuum energy density of the inflaton). In supergravity theories, supersymmetry breaking is transmitted by gravitational interactions and the supersymmetry-breaking mass squared is naturally $C_H H^2$, where H is the Hubble parameter and $C_H = \mathcal{O}(1)$. To illustrate this effect, consider a term in the Kähler potential of the form

$$\delta K = -C_H \int d^4\,\theta \frac{1}{M_{\rm P}^2} I^\dagger I X^\dagger X, \tag{2.7}$$

where I is the field which dominates the energy density ρ of the Universe, that is $\rho \simeq \langle \int d^4\theta I^\dagger I\rangle$. During inflation, I is identified with the inflaton field and $\rho = V(I) = 3H^2 M_{\rm P}^2$. The term (2.7) therefore provides an effective X mass $M_X^2 = 3C_H H^2$. The same mass term will appear during the coherent oscillations of the inflaton field I after inflation. This example can be easily generalized, and the resulting potential of the scalar moduli fields during inflation is of the form $V = H^2 M_{\rm P}^2\, \mathcal{V}(|X|/M_{\rm P})$. We conclude that moduli quanta will be gravitationally produced in the early Universe with a mass squared of the form

$$M_X^2 \simeq m_X^2 + C_H H^2. \tag{2.8}$$

Here m_X accounts for the mass term generated by any possible source of supersymmetry breaking whose F-term is not dominating the energy density of the Universe during inflation, but persists in the zero-temperature limit. For this reason, we expect m_X to be of the order of TeV, the present effective supersymmetry-breaking scale. It is important to bear in mind – though – that the case $C_H = 0$ is certainly a possibility. Indeed, in supergravity models which possess a Heisenberg symmetry, supersymmetry breaking makes no contribution to scalar masses, leaving supersymmetric flat directions unmodified at tree-level. No-scale supergravity of the $SU(N,1)$ type and the untwisted sectors from orbifold compactifications are special cases of this general set of models.[1]

We have numerically integrated eq. (2.3), in presence of an inflaton with quadratic potential, up to the epoch when the X field starts oscillating on all scales and the field fluctuations are transformed into particles which redshift as a^{-3} thereafter. This procedure determines a well-defined quantity, the total number of particles $n_X a^3$, which is constant at late times and is easy to compute numerically. However, the calculation is technically difficult in the regime of very small masses. In this regime a better strategy is represented by adopting observables which are independent of m_X at small m_X. In this limit – and $C_H \to 0$ – the present-day energy density of produced X-particles is an appropriate quantity, since it is independent of m_X (see ref. [11] for a complete discussion of this point). Therefore, for small masses, the number density n_X is better inferred from the ratio ρ_X/m_X. On the other hand, we expect that at $C_H \gtrsim 1$ the number density of produced X-particles will be independent of m_X, since the field fluctuations will enter the oscillating regime at all scales at the epoch in which $M_X^2 \gg m_X^2$. Therefore, in the limit of relatively large C_H, it is more convenient to use n_X.

In our numerical results, we have normalized n_X by the entropy density. In doing so, we have assumed that the reheating temperature after inflation is $T_{RH} = 10^9$ GeV and the mass of the inflaton field is $m_I = 10^{13}$ GeV. The results for scalar fields with minimal coupling ($\xi = 0$) are summarized in fig. 1. This figure shows the limiting behaviour of n_X/s as discussed above, which allows to extract the relevant physical quantities at arbitrarily small m_X. In particular, notice that for small C_H, the ratio n_X/s scales like m_X^{-1}. If the reader wishes to adopt different values for the reheating temperature or the inflaton mass, the data in fig. 1 have to be multiplied by the ratio

$$\left(\frac{T_{RH}}{10^9 \text{ GeV}}\right)\left(\frac{m_I}{10^{13} \text{ GeV}}\right). \tag{2.9}$$

Furthermore, if one wants to include the effect of significant entropy release at some late time after reheating, the data in fig. 1 have to be divided by the amount of entropy increase in the comoving volume, $\gamma \equiv s_{\text{final}}/s_{\text{initial}}$.

From fig. 1 we infer that scalar moduli quanta are very efficiently produced by gravitational effects. In particular, for the realistic case $m_X \sim m_{3/2} \sim 1$ TeV (or $m_X/m_I \simeq 10^{-10}$), the ratio n_X/s turns out to be extremely large for small C_H

$$\frac{n_X}{s} \simeq 5 \times 10^{-5}. \tag{2.10}$$

[1] An analytical study of the gravitational production of scalar moduli in the regime $C_H \ll 1$ can be found in Ref. [2].

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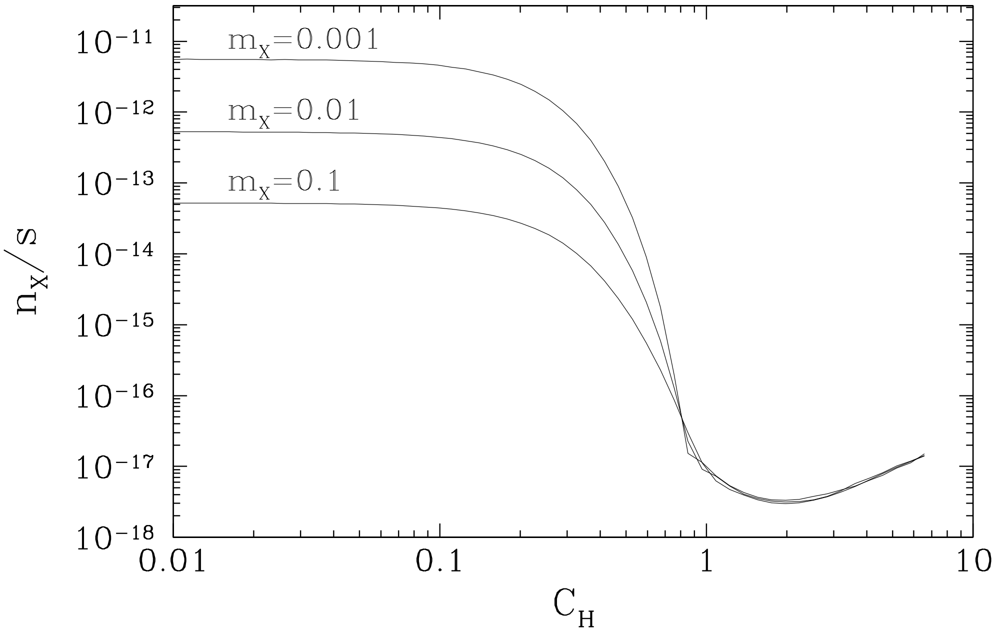

Figure 1: The ratio n_X/s as a function of C_H for scalar moduli particles X minimally coupled to gravity and with squared masses $M_X^2 = m_X^2 + C_H H^2$. In the figure m_X is expressed in units of the inflaton mass m_I, and $T_{RH} = 10^9$ GeV, $m_I = 10^{13}$ GeV.

This result is seven orders of magnitudes above the limit in eq. (1.1). In this case, the non-thermal production is so efficient that the thermal scatterings during reheating become completely irrelevant and the upper bound on the reheating temperature in order to get $n_X/s \lesssim 10^{-12}$ becomes as low as 100 GeV. Only when the parameter C_H approaches unity, regardless of the value of m_X, does the number density of X particles become acceptably small.

Another way of presenting the bound in eq. (1.1) is to compute the energy density in scalar moduli particles normalized to the critical density ρ_c in the Universe, $\Omega_X = \rho_X/\rho_c$. Were these particles stable, the parameter Ω_X would be

$$\Omega_X h^2 \simeq 4 \times 10^{11} \left(\frac{m_X}{\text{TeV}}\right) \left(\frac{n_X}{s}\right), \tag{2.11}$$

where h is the Hubble rate in units of 100 km sec^{-1} Mpc^{-1}. Therefore the bound in eq. (1.1) can be recast into the form

$$\Omega_X h^2 \lesssim 4 \times 10^{-1} \left(\frac{m_X}{\text{TeV}}\right). \tag{2.12}$$

The parameter Ω_X is shown in fig. 2. Again, this figure was calculated assuming $T_{RH} = 10^9$ GeV and $m_I = 10^{13}$ GeV. If one wishes to adopt different values of T_{RH} and m_I or to take into account some entropy release, the data in this figure have to

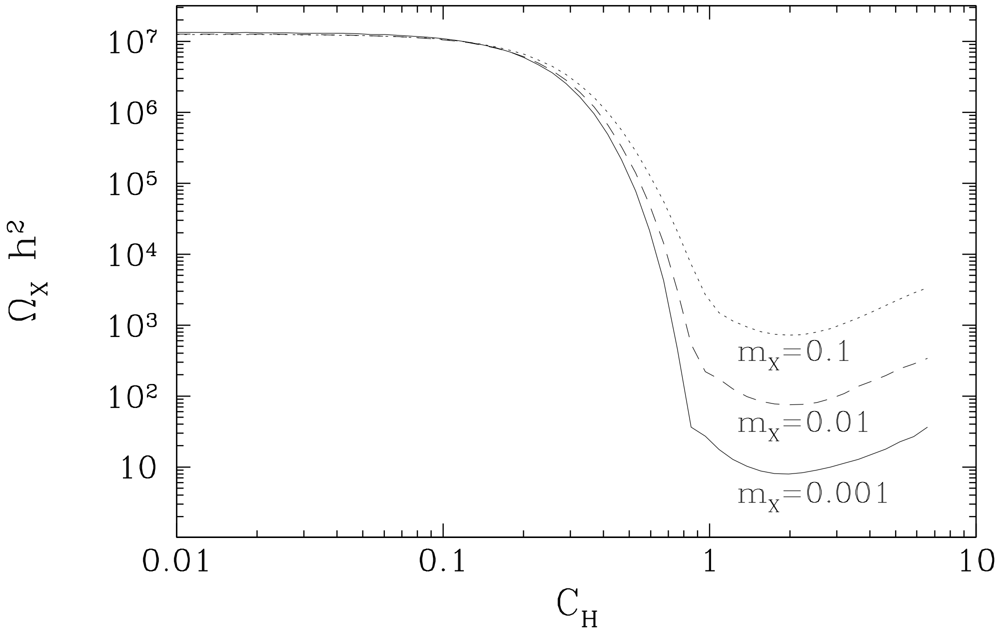

Figure 2: Ratio of the energy density in scalar moduli particles minimally coupled to gravity to the critical density, as a function of C_H. Conventions are the same as in fig. 1.

be multiplied by the factor

$$\gamma^{-1}\left(\frac{T_{RH}}{10^9\ \mathrm{GeV}}\right)\left(\frac{m_I}{10^{13}\ \mathrm{GeV}}\right)^2 . \tag{2.13}$$

Figures 3 and 4 represent the ratio n_X/s and $\Omega_X h^2$ as a function of ξ for $C_H = 0$. From the data one can infer that scalar moduli particle production is extremely dangerous unless ξ is very close to the conformal value $\xi = 1/6$, where particle production shuts off for small m_X.

Our findings are also relevant for the idea of enhanced symmetries and the ground state of string theory [14]. We know that there are often points in the moduli space with maximally enhanced symmetry, i.e. points where all of the moduli are charged under some symmetry (which may be continuous or discrete). These moduli do not suffer from the cosmological moduli problem generated by their large coherent oscillations. For ordinary moduli, finite temperature and/or curvature effects (say during inflation) are likely to leave the Universe in a state which does not correspond to the present minimum of the moduli potential. The Universe remains in this state until the Hubble parameter is of order of the curvature of the potential, after which the system oscillates. The moduli typically dominate the energy density of the Universe when they decay, leading to catastrophic consequences. However, if the vacuum is a point of maximally enhanced symmetry, it is quite natural for the Universe to start out in this state. For example, during inflation, even though the potential for the moduli is modified, it can be expected that the system remains in

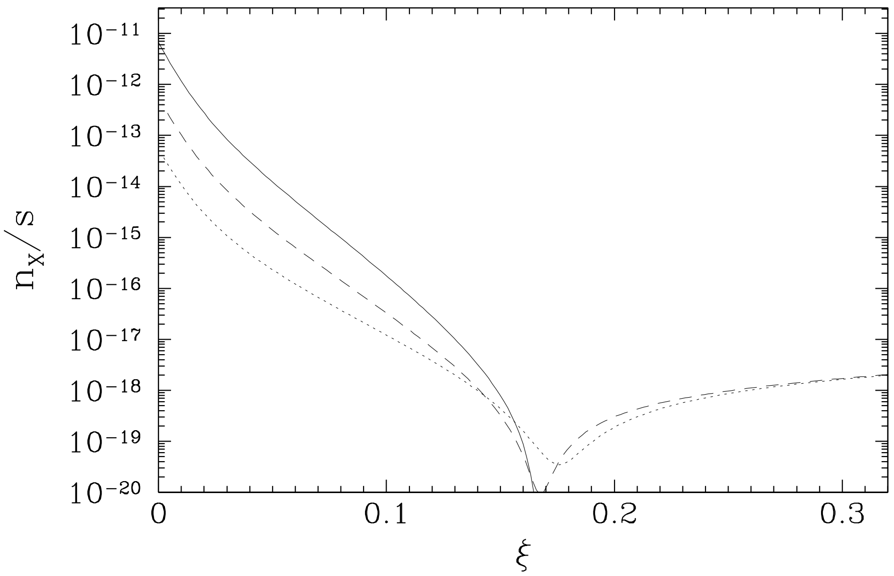

Figure 3: The ratio n_X/s of scalar moduli particles as a function of the parameter ξ, for $C_H = 0$. The solid, long-dashed and short-dashed lines correspond to $m_X/m_I = 0.001$, $m_X/m_I = 0.01$ and $m_X/m_I = 0.1$, respectively.

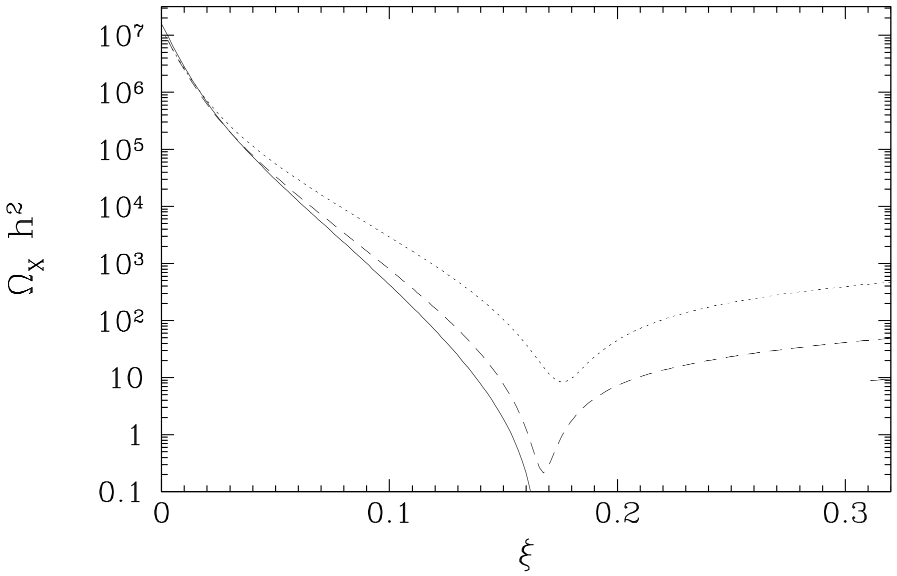

Figure 4: Ratio of the energy density in scalar moduli particles to the critical density as a function of ξ, for $C_H = 0$. The solid, long-dashed and short-dashed lines correspond to $m_X/m_I = 0.001$, $m_X/m_I = 0.01$ and $m_X/m_I = 0.1$, respectively.

the symmetric state. Finite temperature effects also tend to prefer states of higher symmetry [14]. However, this selection rule does not prevent the scalar quanta from being copiously produced by the gravitational effects, unless the effective mass of the scalar excitations around the point of enhanced symmetry is much larger than the Hubble rate during inflation. Therefore, gravitational production of these special moduli fields has to be checked case by case, in order to assess the viability of a model.

3. Non-thermal production of gravitinos

In this section we compute the nonthermal production of gravitinos in timedependent gravitational backgrounds. Before solving the relevant equations of motion in curved space, it is useful to recall the known results of gravitino propagation in flat space.

The free propagation of massive gravitinos in Minkowski space is described by four Majorana spinors ψ^a, satisfying the Rarita-Schwinger equation

$$R^a \equiv \epsilon^{abcd}\gamma_5\gamma_b\partial_c\psi_d + 2m\sigma^{ab}\psi_b = 0. \tag{3.1}$$

Here flat space indices are denoted by Latin letters and are contracted by the metric $\eta = \mathrm{diag}(+,-,-,-)$; we also use the convention

$$\epsilon^{0123} = +1, \qquad \sigma^{ab} = \frac{1}{4}\left[\gamma_a, \gamma_b\right]. \tag{3.2}$$

The mass term in eq. (3.1) explicitly breaks supersymmetry. However, we are implicitly assuming that supersymmetry is spontaneously broken and eq. (3.1) follows from choosing a supersymmetric gauge such that the Goldstino field is fully eliminated from the lagrangean. This choice is analogous to the unitary gauge in Yang-Mills gauge theories.

Using the identity

$$\epsilon^{abcd}\gamma_5\gamma_b = \frac{i}{2}\left(\gamma^a\gamma^b\gamma^c - \gamma^c\gamma^b\gamma^a\right), \tag{3.3}$$

eq. (3.1) becomes

$$R^a = \frac{i}{2}\left(\gamma^a \not\partial\, \gamma\cdot\psi - \gamma^b \not\partial\, \gamma^a\, \psi_b\right) + m\left(\gamma^a\, \gamma\cdot\psi - \psi^a\right) = 0. \tag{3.4}$$

From eq. (3.4) we obtain the following two constraints

$$\gamma\cdot R = 2i\left(\not\partial\, \gamma\cdot\psi - \partial\cdot\psi\right) + 3m\, \gamma\cdot\psi = 0, \tag{3.5}$$

$$\partial\cdot R = m\left(\not\partial\, \gamma\cdot\psi - \partial\cdot\psi\right) = 0. \tag{3.6}$$

For $m \neq 0$, these constraints imply the conditions $\gamma\cdot\psi = 0$ and $\partial\cdot\psi = 0$, which eliminate two spinors out of ψ^a, leaving the appropriate degrees of freedom for the propagation of the spin $\pm 3/2$ and $\pm 1/2$ components. Replacing eqs. (3.5)–(3.6) in eq. (3.4), we obtain that the propagation of physical states obeys the ordinary Dirac equation

$$(i\partial\!\!\!/ - m)\,\psi^a = 0. \tag{3.7}$$

For $m = 0$, eq. (3.1) possesses a gauge symmetry $\delta_\epsilon \psi_a = \partial_a \epsilon$, and we can choose a gauge fixing such that $\gamma \cdot \psi = 0$. Then, eq. (3.5) leads to $\partial \cdot \psi = 0$. However, the choice $\gamma \cdot \psi = 0$ still allows gauge transformation subject to $\partial\!\!\!/ \epsilon = 0$, and a further condition is required to fully fix the gauge. This condition eliminates an additional fermionic component, leaving only the $\pm 3/2$ spin states as physical particles. Notice the complete analogy with the case of spin-one particles, in which the Lorentz condition $\partial \cdot A = 0$ follows from the equation of motion in the massive case and it is a gauge choice in the massless case. Also, for a massless spin-one particle, the gauge fixing requires a further condition, which eliminates the scalar degree of freedom.

Let us turn now to the case of curved space, in which the gravitino equation of motion becomes

$$R^\mu \equiv \epsilon^{\mu\nu\rho\sigma} \gamma_5 \hat{\gamma}_\nu \mathcal{D}_\rho \psi_\sigma = 0. \tag{3.8}$$

Greek letters denote space-time indices and Latin letters denote tangent-space indices. Gamma matrices and the Levi-Civita symbol with curved indices are defined by

$$\hat{\gamma}^\mu \equiv e^\mu_a \gamma^a, \qquad \epsilon^{\mu\nu\rho\sigma} \equiv e e^\mu_a e^\nu_b e^\rho_c e^\sigma_d \epsilon^{abcd}, \tag{3.9}$$

where e^μ_a is the vierbein. $\mathcal{D}_\mu$ is the covariant derivative with respect to the spinorial structure modified to also reproduce the mass term

$$\mathcal{D}_\mu = D_\mu + \frac{i}{2} m \hat{\gamma}_\mu, \qquad D_\mu = \partial_\mu + \frac{1}{2} \omega_{\mu ab} \sigma^{ab}. \tag{3.10}$$

The spin connection $\omega_{\mu ab}$ can be obtained (in first-order formalism) by solving the supergravity equation of motion where the gravitino, the vierbein, and the spin connection are treated as independent fields. One finds (see e.g. ref. [15])

$$\omega_{\mu ab} = \frac{1}{2} \left(-C_{\mu ab} + C_{ab\mu} + C_{b\mu a} \right), \tag{3.11}$$

$$C^a{}_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu - \frac{1}{2M_{\rm P}^2} \bar{\psi}_\mu \gamma^a \psi_\nu, \tag{3.12}$$

where $\bar{\psi}_\mu \equiv \psi^\dagger_\mu \gamma^0$.

For our cosmological considerations, we are interested in the case of spatially flat Friedmann-Robertson-Walker metrics, in which the line element can be written as $ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2)$. Here $\vec{x} = (x^1, x^2, x^3)$ are comoving space coordinates,

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$x^0 \equiv \eta$ is the conformal time and a is the scale factor, such that $a^{-1} = d\eta/dt$. Thus, the vierbein and the metric can be written as $e^a_\mu = a\delta^a_\mu$, $e^\mu_a = a^{-1}\delta^\mu_a$, $g_{\mu\nu} = a^2\eta_{\mu\nu}$, $g^{\mu\nu} = a^{-2}\eta^{\mu\nu}$. In this case, the covariant derivative in eq. (3.10) reduces to

$$\mathcal{D}_\mu = \partial_\mu + \frac{\dot{a}}{4a^2}\left(\hat{\gamma}_\mu\gamma^0 - \gamma^0\hat{\gamma}_\mu\right) + \frac{i}{2}m\hat{\gamma}_\mu, \tag{3.13}$$

where the dot denotes the derivative with respect to conformal time. Here we have dropped from eq. (3.12) the torsion term, bilinear in the gravitino field. This term is crucial for establishing the consistency of supergravity [16]. However, for our considerations, it can be safely ignored since the number density of produced gravitinos is sufficiently small. During the cosmological epochs we will consider, both m and H are much smaller than the Planck mass.

Using the identity in eq. (3.3), we can rewrite eq. (3.8) in the Friedmann-Robertson-Walker metric as

$$\begin{aligned} R^\mu = {} & i\left(\hat{\not\partial}\psi^\mu - \partial^\mu\hat{\gamma}\cdot\psi + \hat{\gamma}^\mu\hat{\not\partial}\hat{\gamma}\cdot\psi - \hat{\gamma}^\mu\partial\cdot\psi\right) + \\ & + i\frac{\dot{a}}{2a}\left(5\hat{\gamma}^0\psi^\mu - 3\hat{\gamma}^\mu\psi^0 + 3\hat{\gamma}^\mu\hat{\gamma}^0\hat{\gamma}\cdot\psi - g^{\mu 0}\hat{\gamma}\cdot\psi\right) + \\ & + m\left(\hat{\gamma}^\mu\hat{\gamma}\cdot\psi - \psi^\mu\right) = 0\,, \end{aligned} \tag{3.14}$$

where $\hat{\not\partial} \equiv \hat{\gamma}^\mu\partial_\mu$ and $\partial\cdot\psi \equiv \partial^\mu\psi_\mu = \partial_\mu\psi^\mu + 2\dot{a}/a\psi^0$. The condition $\mathcal{D}\cdot R = 0$ gives the following algebraic constraint:[2]

$$\begin{aligned} \psi^0 &= c\sum_{i=1}^{3}\hat{\gamma}_i\psi^i\,, \\ c &= \frac{1}{3a\left(\frac{\dot{a}^2}{a^4} + m^2\right)}\left[\left(-2\frac{\ddot{a}}{a^3} + \frac{\dot{a}^2}{a^4} - 3m^2\right)\gamma^0 + 2i\frac{\dot{m}}{a}\right]. \end{aligned} \tag{3.15}$$

Here we have also considered the possibility that the gravitino mass is time dependent, since in general it is determined by time-varying supersymmetry-breaking background fields. In the flat space-time limit with constant gravitino mass, one gets $c = -\gamma^0$ and recovers the gauge fixing condition $\gamma\cdot\psi = 0$.

Since spatial translations generate an exact symmetry of space-time, it is convenient to expand the gravitino field in momentum modes, $\psi^\mu(\eta,\vec{x}) \sim \int d^3k e^{-i\vec{k}\cdot\vec{x}}\psi^\mu_{\vec{k}}(\eta)$. In the following we will consider the equation of motion for a single momentum mode and choose the coordinates such that $\vec{k}$ is along the x^3 direction. For simplicity, we

[2]The parameter c can be expressed in terms of the background energy density ρ and pressure p using the Einstein equations

$$c = \frac{\left(p - 3m^2M_{\rm P}^2\right)\gamma^0 + 2i\dot{m}M_{\rm P}^2}{a\left(\rho + 3m^2M_{\rm P}^2\right)}\,.$$

This expression agrees with the result found in ref. [13].

drop the index $\vec{k}$ from $\psi^\mu_{\vec{k}}$. Because of the antisymmetric properties of the Levi-Civita symbol, the equation $R^0 = 0$ does not contain time derivatives and therefore describes an algebraic constraint on the gravitino momentum modes, which is given by

$$\begin{aligned} \psi^3 &= \left(d - \hat{\gamma}^3\right)\left(\hat{\gamma}_1\psi^1 + \hat{\gamma}_2\psi^2\right), \\ d &= \frac{k}{a^2\left(\frac{\dot{a}^2}{a^4} + m^2\right)}\left(i\frac{\dot{a}}{a^2}\gamma^0 + m\right), \end{aligned} \tag{3.16}$$

where $k \equiv |\vec{k}|$. It is convenient to describe the remaining independent fields with two Majorana spinors $\psi_{1/2}$ and $\psi_{3/2}$, defined such that

$$\psi^0 = \sqrt{\frac{2}{3}}\, c\ \hat{\gamma}_3\ d\ \hat{\gamma}_1\ \psi_{1/2}, \tag{3.17}$$

$$\psi^1 = \frac{1}{\sqrt{6}}\ \psi_{1/2} + \frac{1}{\sqrt{2}}\ \psi_{3/2}, \tag{3.18}$$

$$\psi^2 = \hat{\gamma}^2\hat{\gamma}_1\left(\frac{1}{\sqrt{6}}\ \psi_{1/2} - \frac{1}{\sqrt{2}}\ \psi_{3/2}\right), \tag{3.19}$$

$$\psi^3 = \sqrt{\frac{2}{3}}\left(d - \hat{\gamma}^3\right)\hat{\gamma}_1\ \psi_{1/2}\,. \tag{3.20}$$

We can show that, in the flat limit and on mass-shell, $\psi_{1/2}$ and $\psi_{3/2}$ correspond to the $\pm 1/2$ and $\pm 3/2$ helicity states by explicitly constructing the helicity projectors. Since the gravitino field is built from the direct product of spin 1/2 (spinorial indices) and spin 1 (vectorial indices) states, the helicity projectors can be decomposed using the appropriate Clebsch-Gordan coefficients

$$\mathcal{P}^\mu_{\pm 3/2} = P_{\pm 1/2}P^\mu_{\pm 1}, \tag{3.21}$$

$$\mathcal{P}^\mu_{\pm 1/2} = \sqrt{\frac{1}{3}}P_{\mp 1/2}P^\mu_{\pm 1} + \sqrt{\frac{2}{3}}P_{\pm 1/2}P^\mu_0. \tag{3.22}$$

The helicity projectors acting on spinorial ($P_{\pm 1/2}$) and vectorial ($P^\mu_{\pm 1,0}$) indices are

$$\begin{aligned} P_{\pm 1/2} &= \frac{1}{2}\left(1 \pm i\gamma^1\gamma^2\right), \\ P^\mu_{\pm 1} = \frac{1}{\sqrt{2}}(0, \mp 1, i, 0), &\qquad P^\mu_0 = \frac{1}{m}(k, 0, 0, \sqrt{k^2 + m^2}) \end{aligned} \tag{3.23}$$

It is now easy to verify that, in the flat limit and on mass-shell, eqs. (3.21) and (3.22) project ψ^μ onto $\psi_{1/2}$ and $\psi_{3/2}$, respectively. One can also verify that the normalization chosen in eqs. (3.17)–(3.20) insures that the Rarita-Schwinger lagrangean leads to canonical kinetic terms for the fields $\psi_{1/2}$ and $\psi_{3/2}$.[3]

[3] Notice that the linear combination $\sum_{i=1}^3 \hat{\gamma}_i\psi^i = \sqrt{\frac{2}{3}}\hat{\gamma}_3 d\hat{\gamma}_1\ \psi_{1/2}$ defines the $\pm 1/2$ helicity state adopted in ref. [13]. The states $\psi_{1/2}$ and $\sum_i \hat{\gamma}_i\psi^i$ differ by a time-dependent function.

The equations of motion for the fields $\psi_{1/2}$ and $\psi_{3/2}$ are derived from eq. (3.14),

$$\left[i\gamma^0\partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 - ma + k\gamma^3\right]\psi_{3/2} = 0, \tag{3.24}$$

$$\left[i\gamma^0\partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 - ma + k\left(A + iB\gamma^0\right)\gamma^3\right]\psi_{1/2} = 0, \tag{3.25}$$

$$A = \frac{1}{3\left(\frac{\dot{a}^2}{a^4} + m^2\right)^2}\left[2\frac{\ddot{a}}{a^3}\left(m^2 - \frac{\dot{a}^2}{a^4}\right) + \frac{\dot{a}^4}{a^8} - 4m^2\frac{\dot{a}^2}{a^4} + 3m^4 - 4\frac{\dot{a}}{a^3}\dot{m}m\right], \tag{3.26}$$

$$B = \frac{2m}{3\left(\frac{\dot{a}^2}{a^4} + m^2\right)^2}\left[2\frac{\ddot{a}\dot{a}}{a^5} - \frac{\dot{a}^3}{a^6} + 3m^2\frac{\dot{a}}{a^2} + \frac{\dot{m}}{ma}\left(m^2 - \frac{\dot{a}^2}{a^4}\right)\right]. \tag{3.27}$$

With the field redefinition in eqs. (3.17)–(3.20), the massive Rarita-Schwinger lagrangean has been diagonalized. The $\pm 3/2$-helicity states satisfy the same equation of motion as an ordinary Dirac particle, except for a $5/2$ coefficient replacing the usual $3/2$ in front of the $\dot{a}/a$ term. This coefficient is determined by the field scaling with a, and it can be simply obtained by recalling that the lagrangean density should scale as an energy density, $\mathcal{L} \sim a^{-4}$. Since $\partial_\mu \sim a^0$ and $\partial^\mu \sim a^{-2}$, a simple inspection of the kinetic terms in the corresponding lagrangeans shows that $\phi \sim a^{-1}$, $\psi \sim a^{-3/2}$, and $\psi^\mu \sim a^{-5/2}$ for spin-0, spin-1/2, and spin-3/2 particles, respectively.

The $\pm 1/2$-helicity states, on the other hand, satisfy the more complicated evolution described by eq. (3.25). In the de Sitter limit ($\ddot{a} = 2\dot{a}^2/a$) with constant gravitino mass ($\dot{m} = 0$), the coefficients in eq. (3.25) become $A = \cos\theta$ and $B = \sin\theta$, with $\tan(\theta/2) = \dot{a}/(ma^2)$. In general, however, the time dependendence of the gravitino mass m is related to the evolution of the scale factor. Let us consider the simple case of a single chiral superfield Φ with minimal kinetic terms. The diagonal time and space components of the Einstein equation become

$$\frac{\dot{a}^2}{a^4} = \frac{1}{3M_{\mathrm{P}}^2}\left[V(\Phi) + \left|\frac{d\Phi}{dt}\right|^2\right], \tag{3.28}$$

$$2\frac{\ddot{a}}{a^3} - \frac{\dot{a}^2}{a^4} = \frac{1}{M_{\mathrm{P}}^2}\left[V(\Phi) - \left|\frac{d\Phi}{dt}\right|^2\right]. \tag{3.29}$$

Using the expression for the gravitino mass m in terms of the superpotential W,

$$m = e^{\frac{\Phi^\dagger\Phi}{2M_{\mathrm{P}}^2}}\frac{|W(\Phi)|}{M_{\mathrm{P}}^2}, \tag{3.30}$$

we can write the scalar potential V as

$$V = e^{\frac{\Phi^\dagger\Phi}{M_{\mathrm{P}}^2}}\left[\left|\partial_\Phi W + \frac{\Phi^\dagger W}{M_{\mathrm{P}}^2}\right|^2 - \frac{3|W|^2}{M_{\mathrm{P}}^2}\right] = m^2M_{\mathrm{P}}^2\left[\left|\frac{\dot{m}M_{\mathrm{P}}}{am\frac{d\Phi}{dt}}\right|^2 - 3\right]. \tag{3.31}$$

Replacing eqs. (3.28) and (3.29) in eq. (3.31), we obtain

$$\dot{m}^2 = -\frac{\ddot{a}^2}{a^4} + \frac{\ddot{a}}{a}\left(\frac{\dot{a}^2}{a^4} - 3m^2\right) + 2\frac{\dot{a}^4}{a^6} + 6\frac{\dot{a}^2}{a^2}m^2. \tag{3.32}$$

When this expression for $\dot{m}$ is used in eqs. (3.26) and (3.27), we obtain

$$A^2 + B^2 = 1. \tag{3.33}$$

This condition plays an important rôle in the solution of the equation for the $\pm 1/2$-helicity states.

To proceed, we explicitly write the momentum expansion for $\psi_{1/2}$ in terms of creation and annihilation operators

$$\psi_{1/2}(\eta, \vec{x}) = a^{-5/2}\int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} \sum_{r=1,2}\left[u^r(\eta,\vec{k})a^r_k + v^r(\eta,\vec{k})a^{r\dagger}_{-k}\right], \tag{3.34}$$

where $v^r(\eta,\vec{k}) = u^{rC}(\eta,-\vec{k})$. For simplicity, we now focus on a single polarization state and omit the index r in the following. Choosing a gamma-matrix representation such that

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1}\end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0\end{pmatrix}, \tag{3.35}$$

the spinor $u^r(\eta,\vec{k})$ satisfies the equation of motion (3.25), i.e.

$$\dot{u}_\pm = \mp i m a u_\pm + k(iA \mp B)u_\mp, \tag{3.36}$$

where $u^T = (u_+, u_-)$. The spinor $v(\eta,\vec{k})$ satisfies the same equation of motion. The treatment of $\psi_{3/2}$ is analogous and it is obtained by just setting $A = 1$ and $B = 0$ in the previous equations.

We can now reduce the system (3.36) of first-order differential equations into a second-order differential equation for the function $f = G^{-1/2}u_+$ with $G \equiv A + iB$:

$$\ddot{f} + \left[k^2 + m^2a^2 + i(ma)^{\cdot} - i\frac{\dot{G}}{G}ma + \frac{\ddot{G}}{2G} - \frac{3}{4}\frac{\dot{G}^2}{G^2}\right] f = 0. \tag{3.37}$$

Here we have made use of the property $|G| = 1$, see eq. (3.33). Defining

$$G \equiv e^{2i\int^\eta d\eta\ \omega}, \qquad \Omega \equiv \omega + ma, \tag{3.38}$$

eq. (3.37) can be rewritten as

$$\ddot{f} + \left(k^2 + \Omega^2 + i\dot{\Omega}\right) f = 0. \tag{3.39}$$

An analogous equation has been derived in ref. [13]. Equation (3.39) is identical to the familiar equation for a spin-1/2 fermion in a time-varying background, if Ω is identified with ma. To get a more transparent interpretation of this frequency Ω, it is useful to consider the limit $|\Phi| \ll M_{\rm P}$, as suggested in ref. [13]. Since $m \propto M_{\rm P}^{-2}$, in this limit the function G tends to the expression

$$G = \frac{p - 2i\dot{m}M_{\rm P}^2}{\rho}, \tag{3.40}$$

where $\rho = |d\Phi/dt|^2 + |\partial_\Phi W|^2$ and $p = |d\Phi/dt|^2 - |\partial_\Phi W|^2$. One can now verify that G satisfies the following differential equation

$$\frac{\dot{G}}{G} = -2i\frac{\partial^2 W}{\partial\Phi^2}. \tag{3.41}$$

Therefore, in the limit of $M_{\rm P} \to \infty$ and fixed $|\Phi|$, $\omega \to -\partial_\Phi^2 W$ and the frequency Ω of the oscillations corresponds to the superpotential mass parameter of the Goldstino which is 'eaten' by the gravitino when supersymmetry is broken. Therefore, eq. (3.39) describing the production of helicity-1/2 gravitinos in supergravity reduces, in the limit of $|\Phi| \ll M_{\rm P}$, to the equation describing the time evolution of the helicity-1/2 Goldstino in global supersymmetry and no suppression by powers of $M_{\rm P}$ is present. The frequency of the oscillations Ω depends upon all the mass scales appearing in the problem, namely the Goldstino mass parameter $\partial^2 W/\partial\Phi^2$, the Hubble rate H and the gravitino mass m. The production of the helicity-1/2 gravitino is expected to be dominated by the fastest time-varying mass scale in the problem.

To compute the abundance of helicity-1/2 gravitinos generated during and after an inflationary stage in the early Universe, one needs to discriminate among various supersymmetric inflationary models [17]. A crucial point to keep in mind is that a generic supersymmetric inflationary stage dominated by an F-term has the problem that the flatness of the potential is spoiled by supergravity corrections or, in other words, the slow-roll parameter $\eta = M_{\rm P}^2 V''/V$ gets contributions of order unity. In simple one chiral field models based on superpotentials of the type $W = m\Phi^2/2$ or higher powers in Φ, $W \sim \Phi^n$, supergravity corrections make inflation impossible to start. To construct a model of inflation in the context of supergravity, one must either invoke accidental cancellations [18], or a period of inflation dominated by a D-term [19], or some particular properties based on string theory [20].

Since the theory of production of helicity-1/2 gravitinos looks similar to the case of helicity-1/2 fermions with a frequency Ω, one can use as a guide the recent results obtained in the theory of generation of Dirac fermions during preheating [21]. During inflation, since the mass scales present in the frequency Ω are approximately constant in time, one does not expect a significant production of gravitinos (the number density can be at most $n_{3/2} \sim H_I^3$, where H_I is the value of the Hubble rate during inflation). However, in the evolution of the Universe subsequent to inflation, a large amount of gravitinos can be produced. During the inflaton oscillations, the Fermi distribution function is rapidly saturated up to some maximum value of the momentum k, i.e. $n_k \simeq 1$ for $k \lesssim k_{\rm max}$ and it is zero otherwise. The resulting number

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density is therefore $n_{3/2} \sim k_{\rm max}^3$. The value of $k_{\rm max}$ is expected to be roughly of the order of the inverse of the time-scale needed for the change of the mass scales of the problem at hand. Let us give a realistic example. Consider the superpotential[4]

$$W = S(\kappa\bar{\phi}\phi - \mu^2), \tag{3.42}$$

where κ is a dimensionless coupling of order unity [22]. Here, ϕ and $\bar{\phi}$ are oppositely charged under all symmetries so that their product is invariant. The canonically-normalized inflaton field is $\Phi \equiv \sqrt{2}|S|$. The superpotential (3.42) leads to hybrid inflation. Indeed, for $\Phi \gg \Phi_c = \mu/\sqrt{\kappa}$, $\phi = \bar{\phi} = 0$ and the potential reduces to $V = \mu^4$ plus supergravity and logarithmic corrections [18]. Therefore, in this regime the Universe is trapped in the false vacuum and we have slow-roll inflation. The scale μ is fixed to be around 10^{15} GeV to reproduce the observed temperature anisotropy. Notice that in this period, the Goldstino mass $\partial^2 W/\partial S^2$ is vanishing.

When $\Phi = \Phi_c$, inflation ends because the false vacuum becomes unstable and the fields ϕ and $\bar{\phi}$ rapidly oscillate around the minimum of the potential at $\langle\bar{\phi}\phi\rangle = \mu^2/\kappa$, while the field Φ rapidly oscillates around zero. The time-scale of the oscillations is $\mathcal{O}(\mu^{-1})$. The mass scales at the end of inflation change by an amount of order of μ within a time-scale $\sim \mu^{-1}$. Therefore, one expects $k_{\rm max} \sim \mu$ and $n_{3/2} \sim \alpha\mu^3$, where α summarizes the uncertainty in the estimate. If all the energy residing in the vacuum during inflation is istantaneously transferred to radiation, the reheating temperature would result to be $T_{RH} \sim \mu \sim 10^{15}$ GeV and the ratio $n_{3/2}$ to the entropy density would be $n_{3/2}/s \sim \alpha$. This is not a realistic situation – though – because such a high reheating temperature is already ruled out by considerations about the gravitino generation through thermal collisions, see the bound in eq. (1.4). In a more realistic scenario in which reheating and thermalization occur sufficiently late, the number density of gravitinos decreases as a^{-3} – a being the scale factor – in the post-inflationary scenario, presumably characterized by a matter-dominated Universe. If this is the case, at reheating the final ratio $n_{3/2}$ to the entropy density is

$$\frac{n_{3/2}}{s} \sim \alpha \frac{T_{RH}}{\mu}. \tag{3.43}$$

If $\alpha = \mathcal{O}(1)$, this violates the bound in eq. (1.1) by at least five orders of magnitude even if $T_{RH} \sim 10^9$ GeV and imposes a stringent upper bound on the reheating temperature $T_{RH} \lesssim 1$ TeV. Notice that the non-thermal production is about five orders of magnitude more efficient than the generation through thermal scatterings during the reheating stage, irrespectively of the value of T_{RH}. A similar result may be obtained in the case of D-term inflation.

[4]Our results on gravitino production are based on a simple one chiral superfield model, but we expect similar results to be valid in the case of a theory with more than one superfield.

The ultimate reason for such a copious generation of gravitinos is that the system relaxes to the minimum in a time-scale much shorter than the Hubble time $\sim H_I^{-1}$, since the frequency is set by the height of the potential $V^{1/4} \gg H_I$ during inflation. This is a common feature of realistic models of supersymmetric inflation.

4. Conclusions

In conclusion, we have investigated the non-thermal production in the early Universe of particles with mass in the TeV range and couplings to matter suppressed by powers of $M_{\rm P}$. If produced with too large abundances, the late decays of these gravitational relics may jeopardize the successful predictions of standard big-bang nucleosynthesis. We have shown that scalar moduli – which are generically present in the mass spectrum of supergravity and string theories – may populate the Universe in large amounts as a result of the expansion of the background space-time acting on the vacuum quantum fluctuations of the moduli fields. The resulting number to entropy density ratio depends upon the way these scalar moduli are coupled to gravity and what is their effective mass during and after inflation. We have shown that the generation of these moduli fields poses no problem only if they couple to gravity with ξ close to 1/6, i.e. they are conformally coupled, and if the effective mass squared $C_H H^2$ is comparable or larger than the Hubble rate, i.e. $C_H \gtrsim 1$. On the contrary, if $\xi \neq 1/6$ and $C_H \lesssim 1$, the generation can be so efficient that the standard predictions of nucleosynthesis are safe only if the final reheating temperature is as low as 100 GeV. This upper bound is considerably smaller than the bound of about 10^9 GeV obtained from considerations about the production of scalar moduli through thermal collisions during the reheat stage.

In this paper, we have also studied the non-thermal generation of gravitinos in the early Universe. The spin-3/2 part of the gravitino is excited only in very small amounts, as the resulting abundance is proportional to the small gravitino mass. On the other hand, the spin-1/2 part obeys the equation of motion of a normal helicity-1/2 Dirac particle in a background whose frequency is a combination of the different mass scales at hand: the superpotential mass parameter of the fermionic superpartner of the scalar field whose F-term breaks supersymmetry, the Hubble rate and the gravitino mass. Again, in most realistic models of supersymmetric inflation, the non-thermal production of gravitinos turns out to be much more efficient than their thermal generation during the reheat stage after inflation. We have found, for example, that the ratio $n_{3/2}/s$ for helicity-1/2 gravitinos in generic models of inflation is roughly given by $T_{RH}/V^{1/4}$, where $V^{1/4} \sim 10^{15}$ GeV is the height of the potential during inflation. This generically leads to an overproduction of gravitinos. However, we should stress that the non-thermal gravitino production is quite sensitive to the specific inflationary model. Therefore, it will become an important ingredient in the search for realistic supersymmetric models of inflation.

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